

**An essay on Philosophy of Mathematics
Education: understanding the
cyberspace and foreseeing the
Mathematics Education horizon**

Dr. Maria A.V.Bicudo
São Paulo State University – UNESP
Mathematics Education Program - Rio Claro - Brazil

Posing the essay argument

- A: the scenery:

In the past 30 years, we went from working using computer resources as tools with specific ends in mind, to the notion that, when we develop activities based on the computer and related media, we are actually in side-by-side terms with them, establishing a dialectic cognitive relationship that reorganizes thought.

Posing the essay argument

- B: The argument:

I understand that it is the role of the Philosophy of Mathematics Education to analyze and ponder about this reality, shedding light on the meanings and senses that emerge from works of Mathematics Education authors, especially those who address the Technologies in Mathematics Education in their research .

Focusing the core: conception of reality

- Why?
- As Lévy, many authors who talk about the cyberspace call it *virtual*.
- I ask: When we are with computers and other media, where are we?
- Understanding cyberspace as a worldly reality materialized in the historicity of the *life-world*, can it be virtual? What does virtual mean philosophically? Why is it important to mathematics educators to think about this?

Clearing life-world concept

- Life-world is the translation to English of *Lebenswelt*.
- *It is a phenomenological Husserlian* conception understood as the world taken in its totality; as the place where we realize our experience, interleaved in spatiality and temporality.

Clearing life-world concept

- Life-world is a reality constructed in the historical and cultural movement that brings together the present, the past, and the future.
- It is not a vessel in which we are placed or in which we drop knowledge, theories, etc., as if these were objects in their own empiricism.

Clearing life-world concept

- Rather, Life-world embraces the spatiality and the temporality in which dimensions we live in with others, whether human or not, in which reality we in turn weave using articulated comprehensions, subjectively and intersubjectively that are materialized in available forms and contents.
- What is intersubjectively understood and is kept via the repetition of successful actions gradually forms itself in *objectualities* through the intertwining of senses and meanings expressed through language.

When we are with computers and other media, where are we?

- When we inquire into *where* our attention lies, deliberately, on the spatial question;
- Authors as CASTELLS (2005), LÉVY (2005), TURKLE (1995), LIKAUSKAS (2005), and others, say that this ***where*** is not as the *real one*, but it is as a *virtual one*.

When we are with computers and other media, where are we?

- The authors above understand that there is a space where meetings indeed take place and in spite of that they name it *virtual*, to differentiate it from the *real space*.
- As for time, CASTELLS (2005) refers to the *atemporality* of time, denoting an ambiguity that is common when one is on the verge of not accepting a concept and yet no other notion has unfolded.

The difficulty faced

- To clarify my thoughts: There is a space where meetings occur but it is not real, it is *virtual* and there is time, but it cannot be seen as the time we use to experience it in our daily life, as it is *a-temporal*.
- I gather that the authors may have meant to say a space that is not three dimensional and a time that is not linear or chronologically measurable.
- What is shown is that the concept of time and of space, worked out under the Classical Physics model, does not explain what is happening in cyberspace.

The difficulty faced

- I understand that in the cyberworld, the *where*, does not fit in this Cartesian space.
- In it, we are unable to point to the locations people or ideas meet making an intersection of two axes — space and time — since this *where* unfolds along fast and dynamic connections that branch out to yet more and more unpredictable connections.

How I understand the cyberspace reality

- I understand it is necessary to open up to the concept of space-time as used in Contemporary Physics so as to begin to comprehend the life-world as we experience it today, and to see that cyberspace manifests itself side by side with the physicality of nature .
- That means: we are living *with* it and *in* it. It is one aspect of the life-world.
- I understand cyberspace as real. Why?

Understanding cyberspace as real

- The question: *what is it, the real?* It is a philosophical question and it is an ontological one.
- It is from this perspective that emerge from the realm of philosophical quest, the aspects surrounding the ***virtual***, the reality, the present, the possible, the probable, and potency.
- That question has been addressed in the History of Philosophy since the Greek philosophers

Understanding cyberspace as real

- By invoking those philosophers we see that Aristotle explains the real as a constant movement of potency and act, form and matter. Two are the pairs: *potency and act*, and *form and matter*.

Understanding cyberspace as real

- The philosopher explains the real oscillating between pure potency and pure matter as a constant movement of potency and act, of form and matter.
- ***Potency*** gains existence in the force of the ***act*** and in the specificities of ***matter***, which present themselves as appropriate to be bestowed with a ***form***.

Understanding reality and science and informational screen

- Making an articulation between the conception of real with science and technology GRANGER (1995) studies the Philosophy of Science based on Aristotle's ideas.
- Back then the author develops the concept of *present*, understood as the actualized, and of *non-present*, which includes the virtual, the possible, and the probable, that is, what may happen, but has not happened yet.

Understanding reality and science and informational screen

- Focusing Sciences: Granger considers science as it is today, in the Western World, and asks: what is science about?
- He goes on and answers: it is about the real. This answer refers to the progressive and constant refinement of mathematical, scientific, technical, and technological instrumentation.

Understanding *reality* and science and technology

- His explanations clarify that science contributes to a representation of the real, according to its perspective and methods.
- His findings also show that Mathematics is the ground on which the logic of the Modern Western Science roots itself. Hence the status of Science as the pillar of Exact and Technological Sciences. According to him, reality in Mathematics is virtual in the sense that it does not depend on empirical contents to be shown, though it depends on formal contents.

Understanding reality and science and technology

- He understands Mathematics as virtual, since it works with *forms in general*, being an ontology of forms that are not directly abstracted from the empirical experience. The *forms in general* with which Mathematics works are, therefore, virtual, possible, and probable;

Understanding reality and science and technology

- These forms may actualize themselves in actions (*acts*) triggered and intertwined to the materiality and techniques (*matter*) available, as well as in particular applications that are approximate probabilistic explanations of what is empirically presented, and so on. The connection between the virtual aspect of Mathematics and the empiricism of Natural Sciences is effected through the scientific-theoretical system of references that supports modes of applicability.

Understanding reality and science and technology

- The realized product, therefore, carries the virtual, the possible, and the probable and it is feasible putting together ***form*** (Mathematics and Sciences) and ***matter*** (technological knowledge and equipment) . This complexity is called by Granger ***informational screen***, which sustains the scientific-technological actualization.

Understanding reality and science and technology

- It is not an inflexible screen, which would determine the invariants of actualization, by lodging the acts and the available materiality. The impossibility of completely realizing the virtual of the general form in Mathematics is transcended by the pluralism and multiplicity of possibilities in Natural Science.

The reality of cyberspace

- The reality of cyberspace is a complexity in which the **virtual** (general form of Mathematics), **the possible**, and the **probable** (the scientific-technological apparatus), **the act** (the actualizations triggered by the actions of the people who work with the informational screen) are present.
- The actualization is realized by the **acts** of the people who act according to their own traits, whether they are imaginative, emotive, cognitive, or judgmental, when they operate with the informational screen.

.

The reality of cyberspace

- I understand the constitution of *life-world* as the constitution of the ground, as a network, where we are.
- So, *this is where* we find ourselves at the moment we are with the computer or other media.
- This involves that there is in the spatiality/temporality the *actual* and *non-actual* modes.

The interweaving of anthropological and epistemological aspects

- These aspects are about the way of being and knowing ourselves *at* and *with* the computer and other media.
- When the person is with a computer and other media she/he is not only in the mode of using it because there is a dialectic that supports an exchange between the person and the computer and that accepts the idea of *reorganization of thought* (TIKHOMIROV, 1979) and, to some extent, of dialogue too.

The interweaving of anthropological and epistemological aspects

- The *being-with-the-computer and other media* (Borba & Villareal, 2005) requires us to try to understand its several constituting layers, which intertwine, but do not overlap. As it was said 'The actualization is realized by the **acts**'.
- We should note that the term that expresses that idea is **action**, understood primarily as intentional that is triggered by the subject, effected with the computer when being with it, and therefore, it is an action between subjectivity and the computer, that is, an *inter-action*.

The interweaving of anthropological and epistemological aspects

- In the framework of phenomenological thought *action* is always intentional and is connected with a unit of meaning which is expressed in different languages producing meanings .

The interweaving of anthropological and epistemological aspects

- Question: Does the computer act?
- We do know that it carries out logical operations.
- But, do these operations constitute an action? Operation takes place according to a logical structure predefined by a program. In this dimension of logical operations, can the human being interact-with-the-computer? It certainly can. In what horizon of acting modes does this interaction occur?

The interweaving of anthropological and epistemological aspects

- TIKHOMIROV discusses the theory of reorganization of human thought, taking the computer as mediator. He explains the chain of logical reasoning, formatted as algorithms and translated as computer programs.
- To him reasoning does not disappear, rather it is reorganized. He concludes that *in “artificial intelligence” knowledge is treated formally and has only an external similarity to genuine human knowledge.*

The actualization is realized by the *acts*

- What strikes me is, essentially, that the computer is not an intentional conscience that aims at what it wants to know or any other mode of *being-with*.
- I also clearly perceive that it has a logical structure that allows the inter-action to take place in terms of the comprehensions of language, as a logically structured phenomenon, having its own semantics, and, also, there is this *being-with* to be involved by the intentional conscience in the dialectics of the see-seen (noesis-noema)

The interweaving of anthropological and epistemological aspects

- I perceived that the authors who study Vigotski and followers, such as Tikhomirov, do not open to the existential comprehension of a person who is *being* and whose characteristic is always to be being, as he/she is at present acting and being always while she/he exists.
- I question once again whether the discussion presented affords to solve the complexity sensed in the life-world, in which cyberspace is present. I understand that it does not.

The interweaving of anthropological and epistemological aspects

- To the phenomenological approach, thought and knowledge transcend the purely logical characteristic, understood as the Aristotelian mode that refers to logos as an affirmation of a predicatively expressed rationality as truth.

The interweaving of anthropological and epistemological aspects

- Thought and knowledge also cover the senses, the perception, the cognition, and the judgments that realize themselves in the embodied subjectivity and intertwine with intersubjectivity, constituting objectivity, that is, the knowledge culturally present in the life-world

The modes of dialogue present in the humans-with-computers

- Let us take the logic of computer programs and the modes through which it reveals itself in languages that may be understood by humans-with-the-computer.
- There is thought, understood as problem solving, and there is indeed a human-computer dialogue, as I see it, from this perspective. This is a dialogue seen as an exchange of information and of modes of expressing logical thought as realized in the interaction between both ends of the system.

The modes of dialogue present in the humans-with-computers

- As FIGUEIREDO (2014) poses, the characteristic of the dialogue human-computer is the interactivity that is realized in thought, revealed as the form of human logical thought, and machine, formed by a logical skeleton, which establishes a dialogue.
- This dialogue evolves in making sense of the answer, as expected by the mode of thinking of the person that triggers the action .

The modes of dialogue present in the humans-with-computers

- The computer answers by changing, transforming, moving something that has been sketched and expressed as a formal logical language. This is the making of language.

Mathematics Education realized in the cyberspace

- I understand that the complexity of the reality of cyberspace, here discussed in ontological, epistemological, and anthropological terms is being perceived by authors who study the theme Mathematics Education as realized with the computer and other media.
- Numerous research efforts have been developed to elicit the mode through which teaching and learning of Mathematics take place in cyberspace.

Mathematics Education realized in the cyberspace

- I understand that these efforts contribute, little by little, to elucidate the mode of being in this spatiality and temporality, teaching, learning, and doing Mathematics Education and producing Mathematics

Answering and posing questions about the reality

The reality of the cyberspace is complex. We live *with* and *in* it changing often of focus and context and people with whom we are.

If the cyberspace is supported by the informational screen possible by the scientific knowledge and by the technology, and if it is a reality of the life-world where we live in, what kind of living-experience will be lived in/by the carnal-body?

Answering questions about epistemology

A research made by BARRETO (2014) shows that by doing Mathematics Education chiefly when we are with children in the early education stage, those that have been called digital natives, we can notice they can be comfortable with computers and other media and that they can dialogue in a logical way answering correctly to the activities requested by the teacher when they are working with specific software.

We notice they get involved in several activities at the same time, absorbing information quickly and operating with it.

Answering questions about the existential way of being

- We can also notice that the interest manifested by these children to the activities proposed by teachers depend on the degree of novelty in the activities developed using software. We notice that if the connection is slow, these subjects will start another activity. We notice they get involved in several activities at the same time. So, they manifest a kind of dispersion of attentivity and of focus.
- What becomes evident in the process of change is the way of living temporality: there is no tolerance, and waiting is not accepted.

Answering questions about the existential way of being

- It is showed that time spent waiting is time to be spent doing something else.
- The flow of time is not lived: present, past, and future are not experienced.
- Everything is about the ***now***, the instantaneous present. Places, scenarios, situations are replaced within a click. Spatiality is also experienced with-the-computer and other media, at the speed of a trip.

Posing anthropological questions

How human beings will become in future time, reasoning very quickly and correctly in logical way and existentially being:

- Always, or mainly, at the *now*?
- Not experiencing time and time-lived in its duration?
- Experiencing spatiality at a high speed?

The foreseen horizon for Mathematics Education

- I understand that we are also on the way to understanding ourselves by researching, teaching, learning, producing knowledge, when we are-with-the-computer and other media.
- The complexity of the life-world we live in influences the teaching and learning of Mathematics, the way it is produced, and our own way of being and of being in the world.

My hope

- The philosophical questions presented are like a backdrop, and were briefly addressed, with the intention of placing them in a perspective and affording the advancement of studies and argumentations that lead us to the comprehension of the life-world in which we are.
- I hope we can go further with the discussion about the theme covered.

References

- Barreto, M.F.T. & Nascimento, F.C. Jogos Digitais na Educação Infantil. *IN* Bicudo, M.A.V. (org.) Ciberespaço – Possibilidades que abre ao mundo da educação. São Paulo: Editora Livraria da Física, 2014
- BICUDO, M. A. V., & ROSA, M. (2010). Realidade e Cibernundo: horizontes filosóficos e educacionais antevistos. Canoas: Editora ULBRA.
- BORBA, M. C., VILLARREAL, M. E. (2005). Humans-with-media and the reorganization of mathematical thinking: information and communication technologies, modeling, visualization, and experimentation. New York: Springer Science.
- BORBA, M. C. (2002). O Computador é a Solução: mas qual é o problema? In SEVERINO, A.J., FAZENDA, I.C.A. (Eds.) Formação docente: rupturas e possibilidades. Campinas (SP): Papirus.
- BORBA, M. C. (1999). Tecnologias Informáticas na Educação Matemática e Reorganização do Pensamento. In BICUDO, M. A. V. (Ed.) Pesquisa em Educação Matemática: concepções e perspectivas. São Paulo: Editora UNESP.

References

- CASTELLS, M. A. (2003). A galáxia da Internet: reflexões sobre a internet, os negócios e a sociedade (translated by Maria Luiza X. de Borges). Rio de Janeiro: Jorge Zahar.
- FIGUEIREDO, O.A. A questão do sentido em computação. *IN* Bicudo, M.A.V. (org.) Ciberespaço – Possibilidades que abre ao mundo da educação. São Paulo: Editora Livraria da Física, 2014.
- GRANGER, Gilles G. Le Probable, le Possible et le Virtuel. Paris: Odile Jacob, 1995. KALINKE, M. A., ALMOULOUD, S. A. (2013). A Mudança Da Linguagem Matemática Para A Linguagem Web E As Suas Implicações Na Interpretação De Problemas Matemáticos. Educação Tematática Digital, 15(1), 201-219.
- LÉVY, P. (2005). O que é o virtual? São Paulo: Editora 34.
- LIKAUSKAS, S. (2005). MUD, MOO, RPG e outros bichos. O Globo. 5 Jun 2005. In: MOOsaico: comunidade virtual multilíngüe. Available from <<http://no.moosaico.merg.ulh.as/comunity/press/OGlobro.html>>. Accessed 20 Aug 2006.

References

- LOPES, E. S. (2005). A realidade do virtual. *Psicologia em Revista*, Belo Horizonte PUC Minas 11(17), 96-112. *Bicudo 1 - 6*
- NIESS, M. L. et al. (2009). Mathematics Teacher TPACK Standards and Development Model. *Contemporary Issues in Technology and Teacher Education* 9(1), 4-24. Available from <http://www.citejournal.org/articles/v9i1mathematics1.pdf>. Accessed 27 Nov 2013
- TIKHOMIROV, O. K. (1979). The Psychological Consequences of Computerization. In WERTSCH, J.V. (1979). *The Concept of Activity in Soviet Psychology* (translated from Russian by James V. Wertsch). New York: M.E. Sharpe, Soviet Copyright Agency.
- TURKLE, S. (1995). *A Vida no Ecrã: a Identidade na Era da Internet* (Life on the Screen: identity in the age of the Internet. New York: Touchstone Edition, 1995, translated by Paulo Faria). Lisboa: Relógio D'Água Editores.